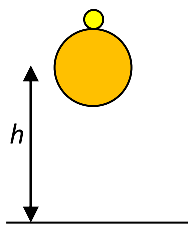
Teacher notes
Topic A

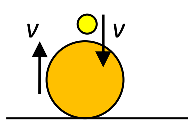
A great demonstration

A tennis ball is placed on top of a basketball and both are dropped at the same time from a height *h* above the ground. We ignore the radii of the balls.

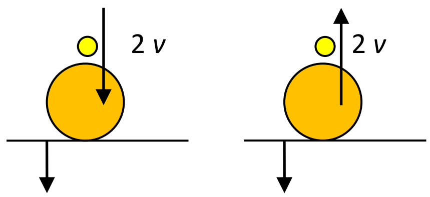


Assuming elastic collisions everywhere, can we predict how high (in terms of *h*) the tennis ball will bounce?

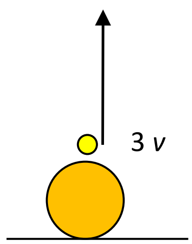
The basketball touches the floor with speed $v=\sqrt{2gh}$ and immediately changes to the same speed but upwards. The speed stays the same because we assumed elastic collisions.



As it starts going up, it collides with the incoming tennis ball which has speed $v=\sqrt{2gh}$ downwards. (The space between the balls in the diagram above is exaggerated.) This collision is also assumed elastic. But the basketball is very much heavier than the tennis ball. For all practical purposes the basketball is like an immovable wall. So it makes sense to look at things from the point of view of the basketball. I.e. we look at things in the reference frame in which the basketball is at rest and the tennis ball is coming down at speed $2v$. If the collision is to be elastic the tennis ball will bounce upwards with a speed $2v$.

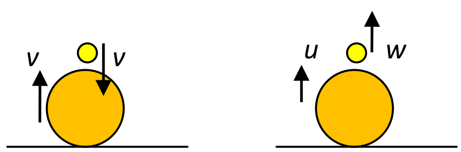


But this is the speed in the reference frame of the basketball. In the lab frame the speed is therefore $3v$.



Hence the tennis ball will rebound to a height of 9*h*.

Without the trick used above we would have to apply conservation of energy and momentum to the situation shown in the diagram.



Momentum: $Mv - mv = Mu + mw$

Energy: $\frac{1}{2}Mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}Mu^2 + \frac{1}{2}mw^2$

These are two equations for the two unknowns *u* and *w*. So solving the first equation for *u* and substituting in the second and after quite a bit of algebra we will find

$w = \frac{(3M - m)v}{M + m}$ and $u = \frac{(M - 3m)v}{M + m}$. Concentrating on the rebound speed, *w*, of the tennis ball:

$$w = \frac{(3M - m)v}{M + m} = \frac{3M(1 - \frac{m}{3M})v}{M(1 + \frac{m}{M})}$$

$$= 3v(1 - \frac{m}{3M})(1 + \frac{m}{M})^{-1} \approx 3v(1 - \frac{m}{3M})(1 - \frac{m}{M})$$

$$\approx 3v(1 - \frac{4m}{3M})$$

Since $M >> m$, we neglect the second term and we get the same approximate answer as before, $w = 3v$.

To see the effect of the masses: the basketball is about 10 times heavier than the tennis ball so in fact the rebound speed is less than $3v$ by a factor of $1-\frac{4}{30}=\frac{13}{15}$. Hence the rebound height will be $9h(\frac{13}{15})^2 \approx 6.8h$ and in reality a bit less because the collisions were never perfectly elastic and we neglected air resistance etc.